ALGONQUIN COLLEGE

Name: Solution

Lab 1 - Separable First Order Differential Equations

1. Solve the given differential equations.
$\sqrt{1+4x^2}\ dy = y^3 x\ dx$

$$\frac{dy}{y^3} = \frac{x\,dx}{\sqrt{1+4x^2}} \Rightarrow y^{-3}dy = (1+4x^2)^{-1/2}x\,dx \Rightarrow \int y^{-3}dy = \int (1+4x^2)^{-\frac{1}{2}}x\,dx \Rightarrow$$

$$\frac{1}{-2}y^{-2} = \frac{1}{8}\int u^{-1/2}du \overset{(*)}{=} \frac{1}{8}\cdot 2u^{1/2} + C = \frac{1}{4}u^{1/2} + C \Rightarrow$$

(*) Let $u = 1+4x^2$
$du = 8x\,dx \Rightarrow$
$\frac{1}{8}du = x\,dx$

$$-\frac{1}{2}y^{-2} = \frac{1}{4}(1+4x^2)^{1/2} + C$$

2. Using the given boundary condition, find the particular solution of the following differential equation.
$x\,dy = y\ln y\,dx, \quad x = 2$ when $y = e$

$$x\,dy = y\ln y\,dx \Rightarrow \frac{dy}{y\ln y} = \frac{dx}{x}. \Rightarrow \int \frac{dy}{y\ln y} \overset{(*)}{=} \int \frac{dx}{x} \Rightarrow$$

$$\int \frac{du}{u} = \ln|x| + C \Rightarrow \ln|u| = \ln|x| + C \Rightarrow$$

(*) Let $u = \ln y$
$du = \frac{1}{y}dy$

$$\ln|\ln y| = \ln|x| + C$$

Sub $x = 2$, $y = e$ in $\ln|\ln y| = \ln|x| + C \Rightarrow$

$$\ln|\ln e| = \ln|2| + C \Rightarrow 0 = \ln 2 + C \Rightarrow C = -\ln 2.$$

Therefore, $\ln|\ln y| = \ln|x| - \ln 2$