ALGONQUIN COLLEGE

Name: Solution

Lab 3 - Linear First Order Differential Equations

1. Given the differential equation $y'\sqrt{x} + \frac{1}{2}y = e^{\sqrt{x}}$

   a. Find the general solution of the differential equation.

Step 1: $y' + \frac{1}{2\sqrt{x}}\, y = \frac{e^{\sqrt{x}}}{\sqrt{x}} \Rightarrow P(x) = \frac{1}{2\sqrt{x}}, \; Q(x) = \frac{e^{\sqrt{x}}}{\sqrt{x}}$

Step 2: $\int P(x)\,dx = \int \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2}\int x^{-1/2}\,dx = \frac{1}{2} \cdot 2x^{1/2} + C = \sqrt{x} + C \Rightarrow$

$I(x) = e^{\int P(x)dx} = e^{\sqrt{x}}$

Step 3: $y \cdot I(x) = \int Q(x) \cdot I(x)\,dx \Rightarrow y \cdot e^{\sqrt{x}} = \int \frac{e^{\sqrt{x}}}{\sqrt{x}} \cdot e^{\sqrt{x}}\,dx \Rightarrow$

$y \cdot e^{\sqrt{x}} = \int \frac{e^{2\sqrt{x}}}{\sqrt{x}}\,dx = \int e^{2x^{1/2}} \cdot x^{-1/2}\,dx$ (*)

(*) Let $u = 2x^{1/2}$, $du = x^{-1/2}dx$

$= \int e^u\,du = e^u + C = e^{2\sqrt{x}} + C \Rightarrow y = e^{\sqrt{x}} + \frac{C}{e^{\sqrt{x}}}$

   b. Find the particular solution of the differential equation which satisfies the boundary condition $x = 1$ when $y = 3$.

Sub $x=1, y=3$ into $y = e^{\sqrt{x}} + \frac{C}{e^{\sqrt{x}}}$:

$3 = e + \frac{C}{e} \Rightarrow 3e - e^2 = C.$

Therefore,

$y = e^{\sqrt{x}} + \frac{3e - e^2}{e^{\sqrt{x}}}$