

Lab 5 - Homogeneous Second Order Differential Equation

1. Solve $y'' + 2y' + y = 0$.

* Aux eq: $r^2 + 2r + 1 = 0 \Rightarrow (r+1)^2 = 0 \Rightarrow r = -1$
double root

* General solution: $y = c_1 e^{-x} + c_2 x e^{-x}$, c_1, c_2 constants

2. Solve $y'' + 4y' + 13y = 0$, $y(0) = 2$, $y(\frac{\pi}{2}) = 1$.

* Aux eq: $r^2 + 4y' + 13 = 0 \Rightarrow r = -2 \pm 3j$

* General solution: $y = e^{-2x} (c_1 \cos 3x + c_2 \sin 3x)$

* Particular solution:

$y(0) = 2 \Rightarrow 2 = e^0 (c_1 \cos 0 + c_2 \sin 0) \Rightarrow 2 = c_1$

$y(\frac{\pi}{2}) = 1 \Rightarrow 1 = e^{-\pi} (c_1 \cos \frac{3\pi}{2} + c_2 \sin \frac{3\pi}{2}) \Rightarrow 1 = -e^{-\pi} c_2 \Rightarrow c_2 = -e^{\pi}$

$\therefore y = e^{-2x} (2 \cos 3x - e^{\pi} \sin 3x)$