

Lab 6 - Nonhomogeneous Second Order Differential Equations

1. Write a trial solution for the given equations.

a. $y'' - 2y = 4x^2 - e^{2x}$

$$y_p(x) = Ax^2 + Bx + C + De^{2x}$$

b. $y'' - 2y' + y = 4x + \sin 3x$

$$y_p(x) = Ax + B + C\cos 3x + D\sin 3x$$

2. Solve the given differential equation using the method of undetermined coefficients.

$$y'' + y' - 2y = 2x$$

Step 1:

Solve $y'' + y' - 2y = 0$

* Aux eq: $r^2 + r - 2 = 0 \Rightarrow (r+2)(r-1) = 0 \Rightarrow r_1 = -2, r_2 = 1$

* GL sol of hom eq: $y_c = c_1 e^{-2x} + c_2 e^x, c_1, c_2$ constants

Step 2:

* Part. sol. of nonhom eq: $G(x) = 2x \Rightarrow$

$$y_p(x) = Ax + B$$

$$y_p'(x) = A$$

$$y_p''(x) = 0$$

Sub y_p, y_p', y_p'' into $y'' + y' - 2y = 2x$:

$$0 + A - 2(Ax + B) = 2x \Rightarrow -2Ax + (A - 2B)x = 2x \Rightarrow$$

$$\begin{cases} -2A = 2 \\ A - 2B = 0 \end{cases} \Rightarrow A = -1, B = -\frac{1}{2} \Rightarrow y_p(x) = -x - \frac{1}{2}$$

Step 3:

* GL sol of nonhom eq: $y = c_1 e^{-2x} + c_2 e^x - x - \frac{1}{2}$

3. Solve the given differential equations using the method of undetermined coefficients.

$$y'' - 3y' - 4y = x + 2\cos x$$

Step 1: Solve $y'' - 3y' - 4y = 0$

* Aux eq: $r^2 - 3r - 4 = 0 \Rightarrow (r-4)(r+1) = 0 \Rightarrow r_1 = 4, r_2 = -1$

* GI sol of hom eq: $y_c = c_1 e^{4x} + c_2 e^{-x}$

Step 2: * Part sol of nonhom eq: $G(x) = x + 2\cos x \Rightarrow$

$$y_p(x) = Ax + B + C\cos x + D\sin x$$

$$y_p'(x) = A - C\sin x + D\cos x$$

$$y_p''(x) = -C\cos x - D\sin x$$

Sub y_p, y_p', y_p'' into $y'' - 3y' - 4y = x + 2\cos x$:

$$-C\cos x - D\sin x - 3(A - C\sin x + D\cos x) - 4(Ax + B + C\cos x + D\sin x) = x + 2\cos x$$

$$-4Ax + (-3A - 4B) + (-C - 3D - 4C)\cos x + (-D + 3C - 4D)\sin x = x + 2\cos x$$

$$\Rightarrow \begin{cases} -4A = 1 & \Rightarrow A = -\frac{1}{4} \\ -3A - 4B = 0 & \Rightarrow -3(-\frac{1}{4}) - 4B = 0 \Rightarrow B = \frac{3}{16} \\ -5C - 3D = 2 & \\ 3C - 5D = 0 & \Rightarrow C = -\frac{5}{17}, D = -\frac{3}{17} \end{cases}$$

$$\therefore y_p = -\frac{1}{4}x + \frac{3}{16} - \frac{5}{17}\cos x - \frac{3}{17}\sin x$$

Step 3: * GI sol of nonhom eq:

$$y = c_1 e^{4x} + c_2 e^{-x} - \frac{1}{4}x + \frac{3}{16} - \frac{5}{17}\cos x - \frac{3}{17}\sin x$$