MAT8103 – Assignment 1 $\frac{\quad}{48}$ Name: Solutions

Due Date:

1) Each of the following integrals can be directly integrated after you have completed a separation of variables. Find the solutions to the following differential equations: (4 marks each)

a) $xdy - \frac{1}{y}dx = 0$

$x dy = \frac{1}{y} dx$

$\int y\,dy = \int \frac{1}{x} dx \Rightarrow \boxed{\frac{y^2}{2} = \ln|x| + c}$

b) $5(x+2)y^4dy + 3dx = 0$

$5(x+2)y^4 dy = -3dx$

$\int 5y^4 dy = \int \frac{-3}{x+2} dx \Rightarrow \cancel{5}\frac{y^5}{\cancel{5}} = -3\ln|x+2| + c$

$u = x+2$

$5\int y^4 dy = -3\int \frac{1}{x+2} dx$

$\boxed{y^5 = -3\ln|x+2| + c}$

c) $3xy^2dy + 4x^3(7 - y^3)dx = 0$

$3xy^2 dy = -4x^3(7-y^3)dx$

$\frac{3y^2 dy}{7-y^3} = \frac{-4x^3 dx}{x} \Rightarrow 3\int \frac{y^2}{7-y^3} dy = -4\int x^2 dx$

$u = 7-y^3$ $\frac{du}{dx} = -3y^2 \Leftrightarrow \frac{du}{-3} = y^2 dx$

$\cancel{3}\int \frac{1}{u} \frac{du}{-\cancel{3}} = -\frac{4x^3}{3} + c$

$\boxed{-\ln|7-y^3| = -\frac{4x^3}{3} + c}$

2) Use integrating combinations to find the solutions of the following differential equations: (4 marks each)

a) $xdy + ydx = 4x^3dx$

$\int d(xy) = \int 4x^3 dx$

$xy = \frac{4x^4}{4} + c \Rightarrow xy = x^4 + c$

$\boxed{y = \frac{x^4 + c}{x}}$

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b) $xdy - ydx = 4x^3dx$

$$\frac{xdy - ydx}{x^2} = \frac{4x^3dx}{x^2}$$

$$\frac{xdy - ydx}{x^2} = 4x\,dx \Rightarrow$$

$$\int d\left(\frac{y}{x}\right) = 4\int x\,dx$$

$$\frac{y}{x} = \frac{4x^2}{2} + C$$

$$\boxed{y = 2x^3 + Cx}$$

c) $xdx + ydy = \frac{2}{y}dy$

$$\int \frac{1}{2}d(x^2+y^2) = 2\int \frac{1}{y}dy$$

$$\boxed{\frac{1}{2}(x^2+y^2) = 2\ln|y| + C}$$

d) $3x^2dx - 4y^2dx - 8xydy = dy$

side

$$d[xy^2] = y^2dx + 2xydy$$

$$3x^2dx - 4(y^2dx + 2xy\,dy) = dy$$

$$\int 3x^2dx - 4\int d[xy^2] = \int dy$$

$$\boxed{x^3 - 4xy^2 = y + C}$$

side

$$d[x^6y^9] = 6x^5y^9dx + 9x^6y^8dy$$

$$= 3x^5y^8(2y\,dx + 3x\,dy)$$

e) $2xy^3dx + 3x^2y^2dy = \frac{1}{3(x^2y^3)^2}dx$

$$xy^2(2ydx + 3xdy) = \frac{1}{3x^4y^6}dx$$

$$3x^5y^8(2ydx + 3xdy) = 1 \cdot dx \Rightarrow \int d[x^6y^9] = \int dx$$

$$\boxed{x^6y^9 = x + C}$$

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3) For the following linear first order differential equations, identify the ~~P~~ $P(x)$ and ~~Q~~ $f(x)$ functions from the question and solve for y. (4 marks each)

a) $dy + \frac{y}{x}dx = dx$

$$\frac{dy}{dx} + \frac{y}{x} = 1$$

$$\frac{dy}{dx} + \underbrace{\left(\frac{1}{x}\right)}_{P(x)} y = \underbrace{1}_{f(x)}$$

Integrating factor: $\mu(x) = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

So,

$$x\frac{dy}{dx} + \frac{y}{x}\cdot x = x$$

$$\int \frac{d[xy]}{dx}dx = \int x\,dx$$

$$xy = \frac{x^2}{2} + c \iff \boxed{y = \frac{x}{2} + \frac{c}{x}}$$

b) $dy + \frac{2y}{x}dx = x^2 \sin(3x^5)\, dx$

$$\frac{dy}{dx} - \underbrace{\left(\frac{2}{x}\right)}_{P(x)} y = \underbrace{x^2 \sin(3x^5)}_{f(x)}$$

Integrating factor: $\mu(x) = e^{\int \frac{2}{x}dx} = e^{2\ln x} = \underbrace{e}_{e^{\ln x^2}} = x^2$

So, $x^2\frac{dy}{dx} - x^2\cdot\frac{2}{x}\cdot y = x^4 \sin(3x^5)$

$u = 3x^5$, $\frac{du}{dx} = 15x^4$

$$\int \frac{d[x^2y]}{dx}dx = \int x^4 \sin(3x^5)\,dx$$

$$x^2y = \frac{1}{15}\int \sin u\, du$$

$$\boxed{x^2y = \frac{-1}{15}\cos(3x^5) + c}$$

c) $\frac{dy}{dx} - y - e^{3x} = 0$

$$\frac{dy}{dx} \underbrace{- y}_{P(x)} = \underbrace{e^{3x}}_{f(x)}$$

Integrating factor: $\mu(x) = e^{-\int 1\,dx} = e^{-x}$

So, $e^{-x}\frac{dy}{dx} - e^{-x}y = e^{-x}e^{3x}$

$u = 2x$

$$\int \frac{d[e^{-x}y]}{dx}dx = \int e^{2x}dx$$

$$\boxed{e^{-x}y = \frac{1}{2}e^{2x} + C}$$

d) $x\frac{dy}{dx} + 3x^3y = x^2e^{-x^3}$

$$\frac{dy}{dx} + \underbrace{3x^2}_{P(x)} y = \underbrace{xe^{-x^3}}_{f(x)}$$

Integrating factor:

$$\mu(x) = e^{\int 3x^2 dx} = e^{x^3}$$

So, $e^{x^3} + x^3 \cdot 3x^2 y = e^{x^3} x e^{-x^3}$

$$\int \frac{d[e^{x^3}y]}{dx}dx = \int x\,dx$$

$$\boxed{e^{x^3}y = \frac{x^2}{2} + c}$$