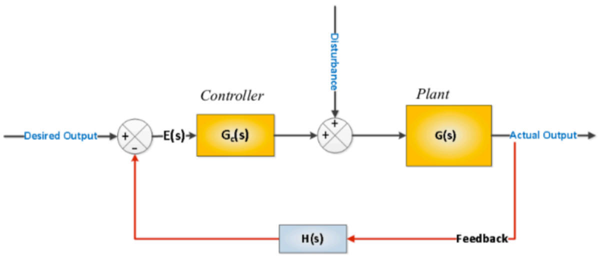
Week5 - Effects of Disturbances

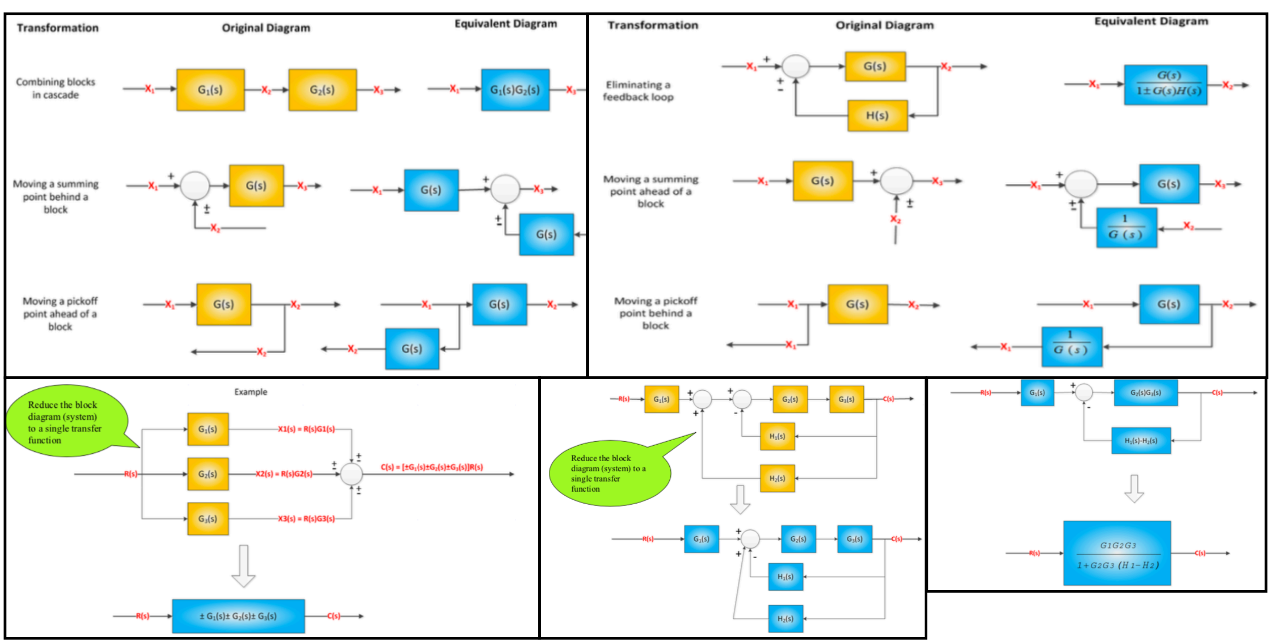
- One of the advantages of the closed-loop systems over the open-loops systems is the ability to reject external disturbances, and measurement noise.
  - External disturbances
    - Are always present (at some degree) in real world applications
    - Must be accounted for in practical closed-loop systems



Closed-loop feedback system with external disturbances

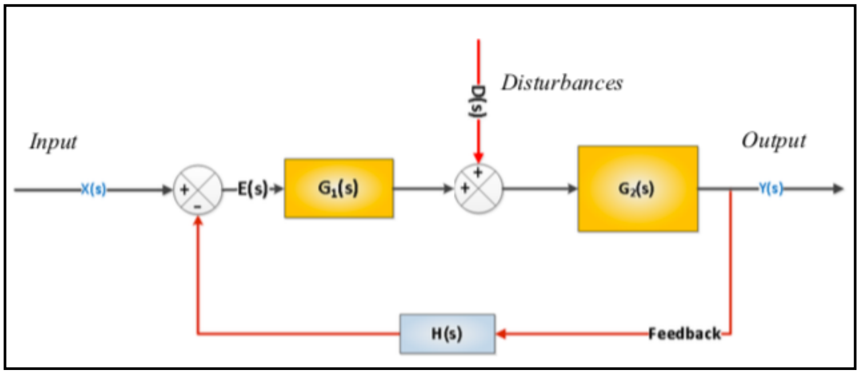
Block Diagram Reduction

- Block diagram reduction techniques
  - Purpose:
    - To be able to re-arrange, reduce, and analyze block diagrams of different systems, involving disturbances (and other) effects
    - To better understand the disturbance effects in control systems
- Blocks can be connected in series only if the output of one block is not affected by the next following block
- If there are loading effects between components it is necessary to combine these components into a single block
- Cascaded block representing non-loading components can be replaced by a single block
- Complicated block diagrams involving many feedback loops can be simplified using step-by-step approach (ie., rearrangement using rules of block diagram algrebra)

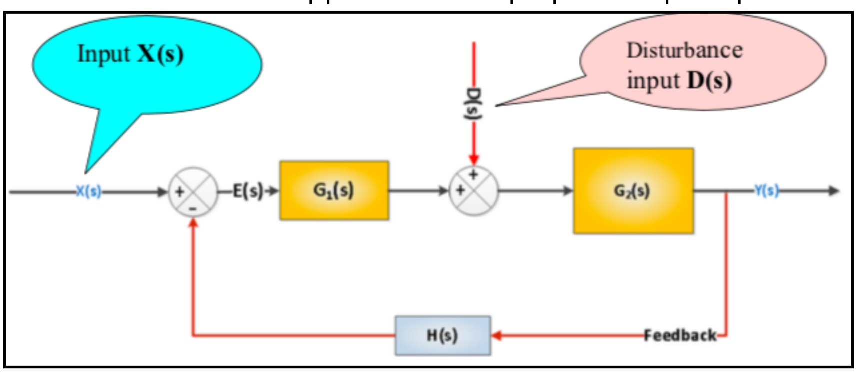


The Effect of Disturbances

- Closed loop system in a presence of disturbances



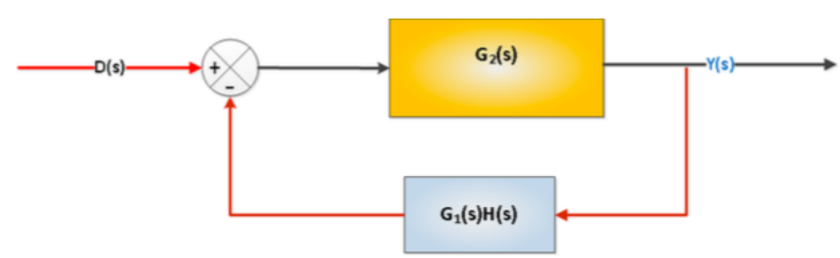
- Expressing the output Y(s) in terms of input X(s) and disturbance D(s) we use ***superposition***
- For linear systems: response to several inputs can be calculated by treating one input at a time and adding the results.
- Let us illustrate the application of superposition principle in this example



- Applying superposition enables us to treat one input at a time:

Response of the system when input X(s) = 0

$$\frac{Y(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$



- The final outcome of the system in the presence of the input X(s) and disturbance D(s) is obtained by adding both responses:

$$Y(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} D(s) + \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)} X(s)$$

$$Y(s) = \left[D(s) + G_1(s)X(s)\right]\left[\frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}\right]$$

Consider the case when:

$$G_1(s)G_2(s)H(s) >> 1 \quad \text{and} \quad G_1(s)H(s) >> 1$$

$$Y(s) = [D(s) + G_1(s)X(s)]\left[\frac{G_2(s)}{G_1(s)G_2(s)H(s)}\right]$$

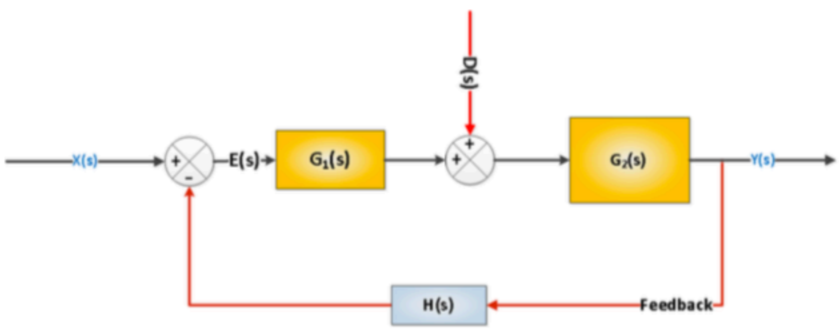
$$Y(s) = [D(s) + G_1(s)X(s)]\left[\frac{1}{G_1(s)H(s)}\right]$$

$$Y(s) = [D(s) + G_1(s)X(s)]\left[\frac{1}{G_1(s)H(s)}\right] = X(s)\frac{1}{H(s)}$$

Negligible since $G_1(s)H(s) >> 1$

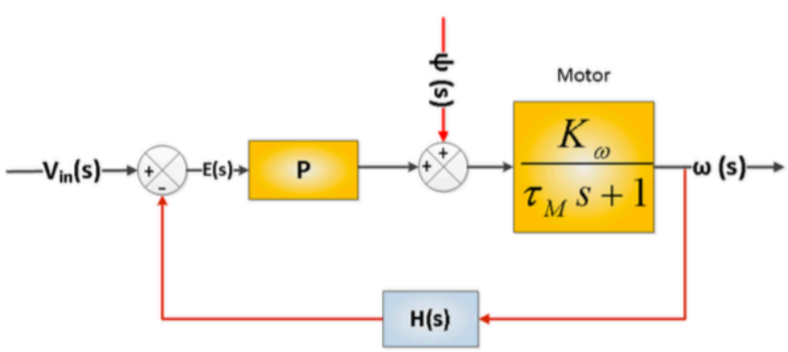
*When **H(s) = 1** we can see that the output **Y(s)** follows the input **X(s)**... and The effect of the disturbance **D(s)** is supressed!*

Example:



Let's adapt the general example of the closed-loop system in a presence of disturbances to a DC motor...

$$X(s) = V_{in}(s) \quad Y(s) = \omega(s) \quad D(s) = \psi(s) \quad G_1 = P \quad G_2 = \frac{K_\omega}{\tau_M s + 1}$$



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$$Y(s) = \left[D(s) + G_1(s)X(s)\right]\left[\frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}\right]$$

$$\omega(s) = \left[\Psi(s) + PV_{in}(s)\right]\left[\frac{\dfrac{K_\omega}{\tau_M s + 1}}{1 + P\dfrac{K_\omega}{\tau_M s + 1}H(s)}\right]$$

$$\omega(s) = \left[\Psi(s) + PV_{in}(s)\right]\left[\frac{K_\omega}{\tau_M s + 1 + PK_\omega H(s)}\right]$$

- Final value of the output is reached when
  - t→ ∞ or s→0, thus we have:

$$\lim_{s\to 0}\omega(s) = \lim_{s\to 0}\left[\Psi(s) + PV_{in}(s)\right]\left[\frac{K_\omega}{\underbrace{\tau_M s}_{0} + 1 + PK_\omega H(s)}\right]$$

$$\omega(s)_{Steady-state} = \left[\Psi(s) + PV_{in}(s)\right]\left[\frac{K_\omega}{1 + PK_\omega H(s)}\right]$$

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Disturbance effect   *SetPoint effect*

$$\omega(s)_{Steady-state} = \Psi(s)\frac{K_\omega}{1 + PK_\omega H(s)} + V_{in}(s)\frac{PK_\omega}{1 + PK_\omega H(s)}$$

Final outcome of a DC motor system in the presence of disturbances

- Let's analyze the final outcome of a DC motor system in the presence of input V_{in} and disturbances ψ

Disturbance effect *SetPoint effect*

$$\omega(s)_{Steady-state} = \Psi(s)\frac{K_\omega}{1+PK_\omega H(s)} + V_{in}(s)\frac{PK_\omega}{1+PK_\omega H(s)}$$

As proportional gain P or H is increased → ψ *decreases*
ψ becomes negligible, however is never completely eliminated

Remarks:

- In order to compensate for the effect of disturbances, we can compensate by adding an offset to the SetPoint, whereas the offset amount is such as to cancel the disturbance effect…
  - The above offset technique will *not work* in the presence of changing (variable) disturbances
  - Use of better (more efficient) techniques shall be investigated at a later time