ALGONQUIN COLLEGE | Algonquin Centre for Construction Excellence

ENG8344E – CONTROL SYSTEMS

Pole Cancellation

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Architecture, Civil, Electrical and Building Science

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Learning Outcome

- Identify the zeros and poles of a control system
- Describe and demonstrate the pole cancellation principles and their use in control system analysis

Outline

- Introduction
- Pole cancellation
- Lowering the order of a system using pole cancellation
- Remarks

Introduction

- Many techniques enable us to evaluate the output response of a system i.e., differential equations, inverse Laplace transform
  - Can be long and time consuming
- Use of poles and zeros to evaluate the output response is a qualitative (fast) technique
  - Simplifies the evaluation/analysis of system's response

Introduction

- The performance of feedback systems is directly related to the location of the closed-loop poles.
- What are the poles of a control system?
  - Values of Laplace transform variable **s** which causes the transfer function to become infinite
    - These are the roots of the denominator's transfer function

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Introduction

- What are the zeros of a control system?
  - Values of Laplace transform variable **s** which causes the transfer function to become zero
    - These are the roots of the numerator's transfer function

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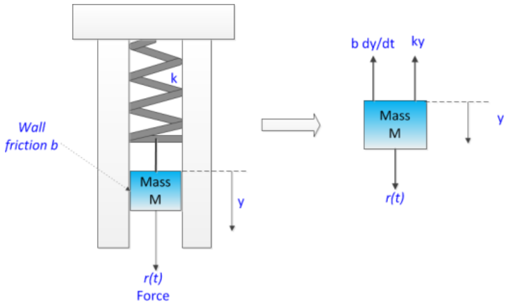
Introduction

- Let us see the usefulness of Laplace transform, and demonstrate the pole and zero concept via an example!



Based on the forces acting on the mass M, and the second Newton's law, we obtain the following differential equation:

$$M\frac{d^2y(t)}{dt^2}+b\frac{dy(t)}{dt}+ky(t)=r(t)$$

Spring mass damper system and its free body diagram

Introduction

- In order to get the response y as a function of time, we need to find the Laplace transform of the equation:

$$\mathcal{L}\left\{M\frac{d^2y(t)}{dt^2}+b\frac{dy(t)}{dt}+ky(t)=r(t)\right\}$$

$$M(s^2Y(s)-sy(0)-\frac{dy(0)}{dt})+b(sY(s)-y(0))+kY(s)=R(s)$$

When $r(t)=0$, and $y(0)=y_0$ and $\left.\frac{dy(t)}{dt}\right|_{t=0}=0$

$$Ms^2Y(s)-Msy_0+bsY(s)-by_0+kY(s)=0$$

Introduction

- Solving the equation $Ms^2Y(s) - Msy_0 + bsY(s) - by_0 + kY(s) = 0$ for Y(s), we get:

$$Y(s) = \frac{(Ms+b)y_0}{Ms^2 + bs + k} = \frac{z(s)}{p(s)}$$

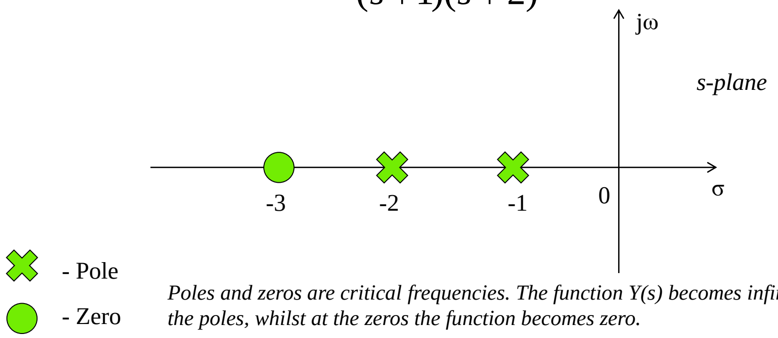
The denominator p(s) when set equal to zero, i.e., p(s)=0 it is called the <u>characteristic equation</u>. The roots of this equation determine the character of the time response; they are called the POLES of a systems.

The roots of the numerator z(s) are called the ZEROS of the system.

Introduction

- Considering a specific case, e.g. when k/M=2 and b/M=3, Y(s) becomes:

$$Y(s) = \frac{(s+3)y_0}{(s+1)(s+2)}$$



Poles and zeros are critical frequencies. The function Y(s) becomes infinite at the poles, whilst at the zeros the function becomes zero.

Introduction

- Expanding Y(s) in a partial expansion we obtain:

$$Y(s) = \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)}$$

Evaluating k_1 and k_2 when $y_0 = 1$, we obtain $k_1 = 2$ and $k_2 = -1$, and thereof:

$$\textbf{y(t)} = \mathcal{L}^{-1}\left\{ \frac{2}{(s+1)} + \frac{-1}{(s+2)} \right\} = 2e^{-t} - e^{-2t}$$

Note: *Final result was obtained via Laplace transform* $e^{-at}u(t) \longleftrightarrow \frac{1}{s+a}$

Introduction

- The relative stability and the transient response of a closed-loop control system are directly related to the location of the roots in the s-plane.

Pole cancellation

- Pole cancellation effects the denominator of the system’s transfer function
  - Hence, reduces the order of the transfer function by cancelling the poles
  - Mathematical order of a system is reduced thus, it becomes easier (less complex) to analyze
- **Note:** *This approach can be used only if the plant transfer function is well known and stable over time.*

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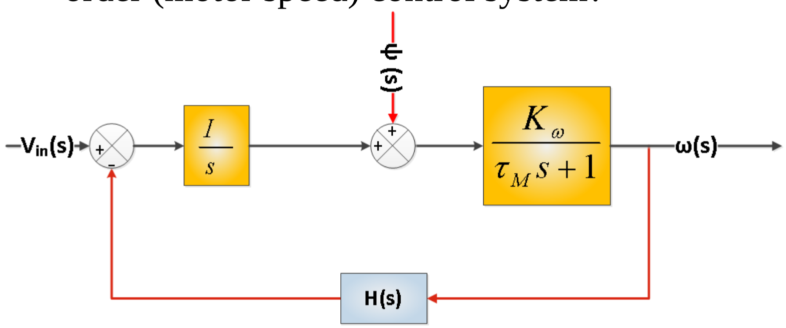
Pole cancellation

- Look at the block diagram below…
  - What can we do to cancel any pole of this first order (motor speed) control system?



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Pole cancellation

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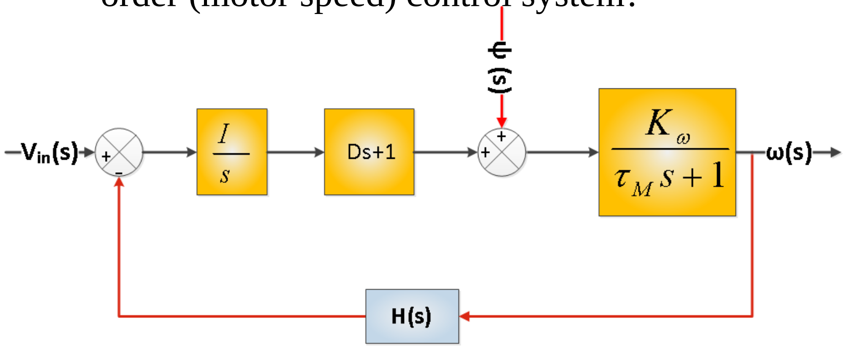

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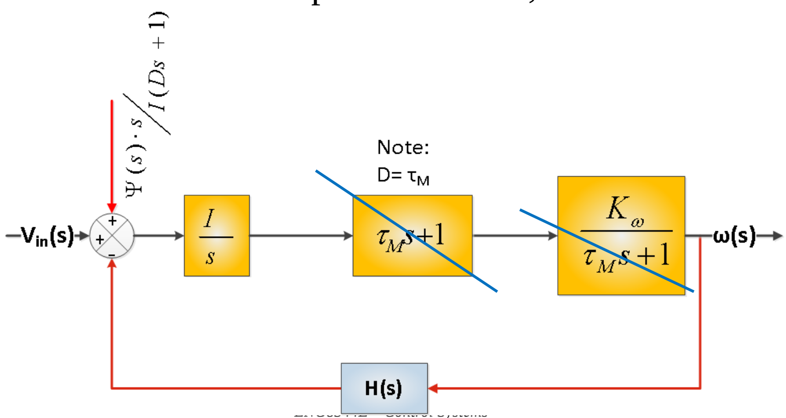
Pole cancellation

- See the **Ds+1** term is tuned (**D = τ_M**) to match the denominator of the plant → hence, it cancels it out



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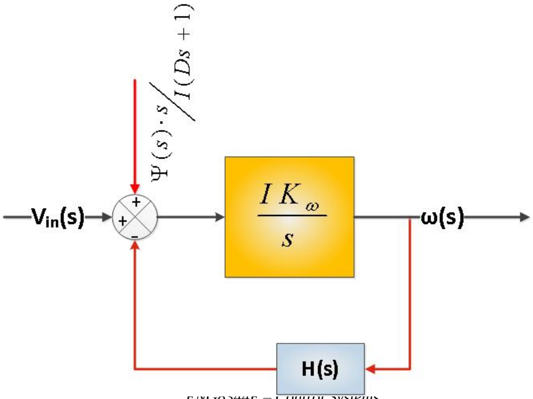

Pole cancellation

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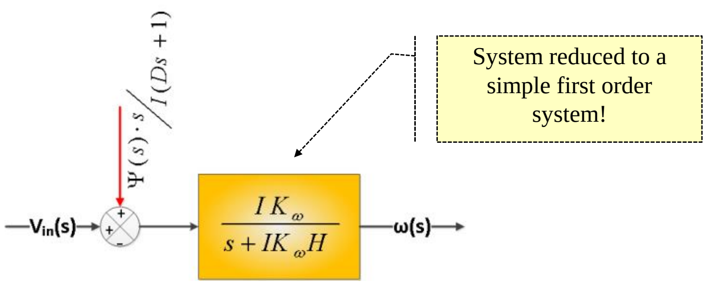

$$\omega = Vin\,\frac{I K_\omega}{s + IK_\omega H} + \Psi(s)\cdot s\,\frac{K_\omega}{Ds^2 + (1 + DI K_\omega H)s + I K_\omega H}$$

Pole cancellation

$$\omega = Vin\,\underbrace{\frac{I\,K_\omega}{s+IK_\omega H}}_{\textit{Setpoint effect}} + \Psi(s)\cdot s\,\underbrace{\frac{K_\omega}{D\,s^2+(1+DI\,K_\omega\,H)\,s+I\,K_\omega\,H}}_{\textit{Disturbance effect}}$$

Note:
- ✓ **I term** can be used to control the response time of a system
- ✓ **H term** can be used to control the gain of a system

Note:
- ✓ Disturbance effect goes to zero as s→0 (steady-state)
- ✓ **D term** cannot be used since it is used to cancel the pole of a system

19

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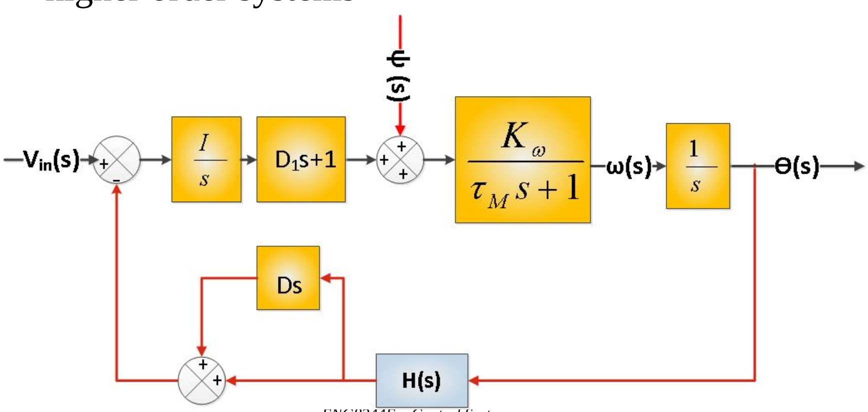
Lowering the order of a system using pole cancellation

- The pole cancellation technique, can also be used for higher order systems



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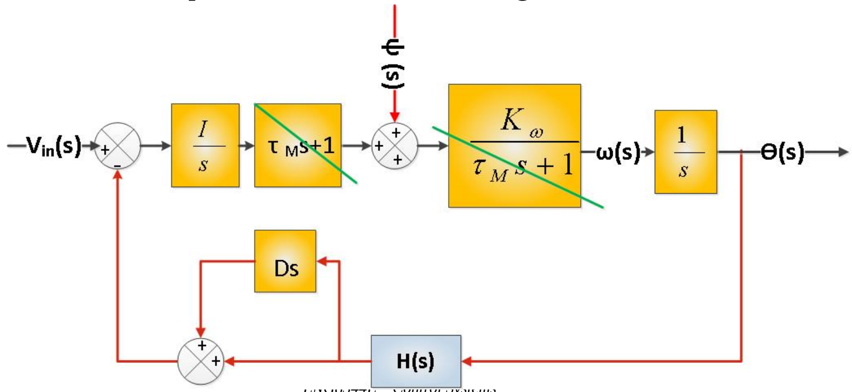
20

Lowering the order of a system using pole cancellation

- See the $\mathbf{D_1 s+1}$ term is tuned $\mathbf{D_1 = \tau_M}$ to match the denominator of the plant → to cancel the pole (reducing the order of a system); and Ds in the feedback path added to control settling time.



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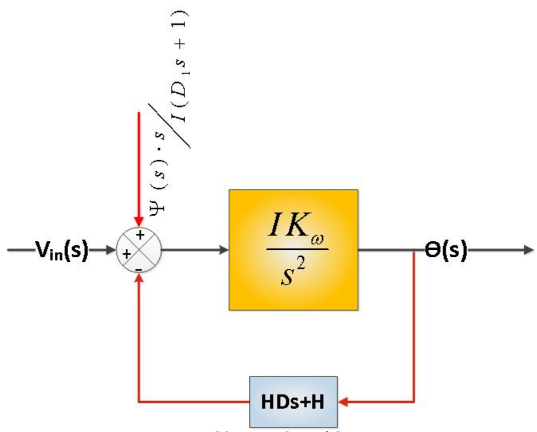
Lowering the order of a system using pole cancellation

- Simplifying further…by using block reduction technique...



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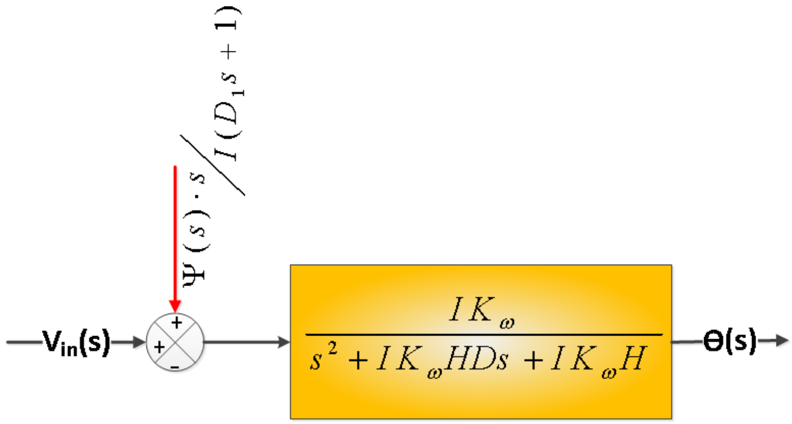
Lowering the order of a system using pole cancellation

- Simplifying further-closed loop transfer function yields…



Lowering the order of a system using pole cancellation

- Using superposition, we get:

$$\theta = Vin\underbrace{\frac{IK_\omega}{s^2+I\,k_\omega\,HDs+I\,k_\omega\,H}}_{\textit{Setpoint effect}}+\underbrace{\frac{\Psi(s)\cdot s}{\cancel{I}_1(D_1 s+1)}\,\frac{\cancel{I}K_\omega}{s^2+I\,k_\omega\,HDs+I\,k_\omega\,H}}_{\textit{Disturbance effect}}$$

Note: Second order system…
- ✓ **I and D term** can be used to control the response time and damping
- ✓ **H term** can be used to control the gain

Note:
- ✓ Disturbance effect goes to zero as s→0 (steady-state)
- ✓ Third order denominator; complicated disturbance response;
- ✓ Large I values can be used to attenuate the amplitude and duration of transients

Pole cancellation

- For both models shown in the pole cancellation section…to reflect reality, the models input must include limiters (at input of plant → to simulate practical maximum values, which might be applied)
- In both cases, if limiter clips the input drive too much the cancellation technique would not be exact
  - This would could result in a loss of control i.e., loss of stability of the system

Remarks

- Use of poles and zeros to evaluate the output response is a qualitative (fast) technique
  - ✓ Simplifies the evaluation/analysis of system's response
- Pole cancellation reduces the order of the transfer function by cancelling the poles
  - ✓ Mathematical order of a system is reduced → it becomes easier to analyze
- The pole cancellation technique can be used only if the plant transfer function is well known and stable over time

References

- Professor Philippe Déziel's notes
- Modern Control Systems, Richard C. Dorf, Robert H. Bishop, 11th edition, Prentice Hall.
- Bateson, Robert N. Introduction to Control System Technology, 6th edition, Prentice Hall.

Additional Information

- The **root locus** method for finding the roots of the characteristic equation was introduced by W.R. Evans is utilized extensively in control engineering practice.
- Root locus is powerful tool for designing and analyzing feedback control systems.

Additional Information

- The **root locus** is a graphical technique which sketches the locus of roots in the s-plane as a parameter is varied
  - Provides a measure of the sensitivity of the roots to parameter variations
- The root locus can be used to obtain quickly qualitative information about the stability and performance of the system

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